KUWAIT UNIVERSITY

Department of Mathematics & Computer Science

Math 102 — July 5, 2008

Calculus II — First Midterm Exam — Time: 90 minutes

No calculators or cell phones are permitted during the exam

Q1. Let $f(x) = \ln x + \dfrac{1}{\ln x}$. [5 pts]

i. Find the domain of f.

ii. Show that f is one-to-one in the interval (e, ∞).

iii. Find the slope of the tangent line to the graph of $f^{-1}(x)$ at the point $P(\frac{5}{2}, e^2)$.

Q2. [4 pts]

i. Evaluate $\sec\left(\sin^{-1}\frac{1}{3}\right) - \sin(\sec^{-1} 3)$.

ii. Show that $\log_2 x = 3\log_8 x$ for all $x > 0$.

Q3. Evaluate $\displaystyle\lim_{x\to 0^+}\left(\frac{1}{x} - \csc x\right)$ if the limit exists. [3 pts]

Q4. Use logarithmic differentiation to find y' given [4 pts]

$$y = \frac{\sinh x \tan^{-1} x}{\sqrt[3]{\cosh x \cos^{-1} x}}$$

Q5. Evaluate the following integrals: [9 pts]

i. $\displaystyle\int \frac{1}{\sqrt[3]{\cosh x - \sinh x}}\,dx$

ii. $\displaystyle\int \frac{1}{x + \ln x^x}\,dx$

iii. $\displaystyle\int \frac{1}{\sqrt{9^{-x} - 1}}\,dx$

MIDTERM 1 SOLUTIONS

Q1. Let $f(x) = \ln x + \dfrac{1}{\ln x}$.

i. Domain of $f = (0,1) \cup (1,\infty)$.

ii. f is increasing in the interval (e,∞) since $x > e$ implies $\ln x > 1$ and we have

$$f'(x) = \frac{1}{x} - \frac{1}{x\,(\ln x)^2} = \frac{1}{x}\left[1 - \frac{1}{(\ln x)^2}\right] > 0$$

iii. $$\frac{d}{dx} f^{-1}(x)\bigg|_{x=5/2} = \frac{1}{f'(e^2)} = \frac{4e^2}{3}$$

Q2. (i) $\dfrac{3}{\sqrt{8}} - \dfrac{\sqrt{8}}{3} = \dfrac{1}{6\sqrt{2}}$.

(ii) $\text{rhs} = 3\,\log_8 x = 3\,\dfrac{\ln x}{\ln 8} = 3\,\dfrac{\ln x}{3\,\ln 2} = \dfrac{\ln x}{\ln 2} = \log_2 x = \text{lhs}.$

Q3. This is of the form $\infty - \infty$, so as $x \to 0^+$:

$$\frac{1}{x} - \csc x = \frac{1}{x} - \frac{1}{\sin x} = \frac{\sin x - x}{x\,\sin x} \to \frac{\cos x - 1}{\sin x + x\,\cos x} \to \frac{-\sin x}{2\,\cos x - x\,\sin x} \to 0$$

Q4.

$$\ln y = \ln\sinh x + \ln\tan^{-1} x - \frac{1}{3}\left[\ln\cosh x + \ln\cos^{-1} x\right]$$

$$\frac{y'}{y} = \coth x + \frac{1}{(\tan^{-1} x)\,(1+x^2)} - \frac{1}{3}\left[\tanh x - \frac{1}{(\cos^{-1} x)\,\sqrt{1-x^2}}\right] \cdots \text{ etc.}$$

Q5. (i) Use $\cosh x - \sinh x = \bar{e}^{x}$ to get $\int e^{+x/3}dx = +3\,e^{+x/3} + c$

(ii) $x + \ln x^x = x\,(1 + \ln x)$ and put $u = 1 + \ln x$ etc.

(iii) Put $u = 3^x$ etc.